Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*4325784148*

**MATHEMATICS** **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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**1** Find the value of $x$ for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where $a$ and $b$ are integers. [4]

**2** Solve the inequality $|3x - a| > 2|x + 2a|$, where $a$ is a positive constant. [4]

**3** **(a)** Given the complex numbers $u = a + \mathrm{i}b$ and $w = c + \mathrm{i}d$, where $a$, $b$, $c$ and $d$ are real, prove that $(u + w)^* = u^* + w^*$. [2]

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**(b)** Solve the equation $(z + 2 + \mathrm{i})^* + (2 + \mathrm{i})z = 0$, giving your answer in the form $x + \mathrm{i}y$ where $x$ and $y$ are real. [4]

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**4** Express $\dfrac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions. [5]

**5** **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers $z$ satisfying the inequalities $|z - 3 - 2\mathrm{i}| \leqslant 1$ and $\operatorname{Im} z \geqslant 2$. [4]

**(b)** Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

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**6** **(a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$ [3]

**(b)** Hence show that $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x \, dx = \frac{1}{5}(3 - \sqrt{2})$. [3]

**7** The variables $x$ and $y$ satisfy the differential equation

$$e^{2x}\frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for $y$ in terms of $x$. [7]

**8** **(a)** By first expanding $\left(\cos^2\theta + \sin^2\theta\right)^2$, show that

$$\cos^4\theta + \sin^4\theta \equiv 1 - \tfrac{1}{2}\sin^2 2\theta.$$

[3]

**(b)** Hence solve the equation

$$\cos^4\theta + \sin^4\theta = \tfrac{5}{9},$$

for $0° < \theta < 180°$. [4]

**9** The equation of a curve is $ye^{2x} - y^2e^x = 2$.

**(a)** Show that $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$. [4]

**(b)** Find the exact coordinates of the point on the curve where the tangent is parallel to the $y$-axis. [4]

**10** With respect to the origin $O$, the position vectors of the points $A$ and $B$ are given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$.

**(a)** Find a vector equation for the line $l$ through $A$ and $B$. [3]

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**(b)** The point $C$ lies on $l$ and is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$.

Find the position vector of $C$. [2]

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**(c)** Find the possible position vectors of the point $P$ on $l$ such that $OP = \sqrt{14}$. [5]

**11** The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leqslant x < \frac{1}{2}\pi$.

**(a)** Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

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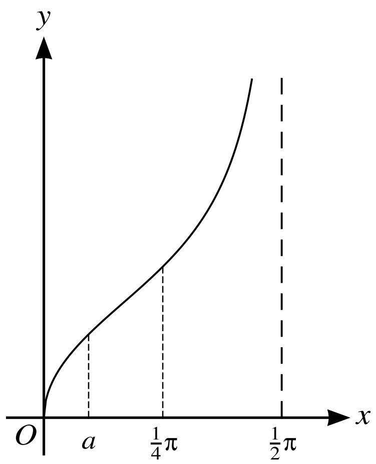
The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.




**(b)** Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

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(c) Use the iterative formula

$$a_{n+1} = \tan^{-1}\left(\tfrac{1}{3}\left(1 - \tan^2 a_n - \tan^3 a_n\right)\right)$$

to determine $a$ correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.